How to calculate the nth term of an arithmetic sequence

Explicit formulas can be used to determine the number of terms in a finite arithmetic sequence. We need to find the common difference, and then determine how many times the common difference must be added to the first term to obtain the final term of the sequence. How To: Given the first three terms and the last term of a finite arithmetic sequence, find the total number of terms. Find the common difference [latex]d[/latex]. Substitute the common difference and the first term into [latex]{a}_{n}={a}_{1}+d\left(n - 1\right)[/latex]. Substitute the last term for [latex]{a}_{n}[/latex] and solve for [latex]n[/latex]. Find the number of terms in the finite arithmetic sequence. [latex]\left\{8\text{, }1\text{, }-6\text{, }...\text{, }-41\right\}[/latex] The common difference can be found by subtracting the first term from the second term. [latex]1 - 8=-7[/latex] The common difference is [latex]-7[/latex] . Substitute the common difference and the initial term of the sequence into the [latex]n\text{th}[/latex] term formula and simplify. [latex]\begin{array}{l}{a}_{n}={a}_{1}+d\left(n - 1\right)\hfill \\ {a}_{n}=8+-7\left(n - 1\right)\hfill \\ {a}_{n}=15 - 7n\hfill \end{array}[/latex] Substitute [latex]-41[/latex] for [latex]{a}_{n}[/latex] and solve for [latex]n[/latex] [latex]\begin{array}{l}-41=15 - 7n\hfill \\ 8=n\hfill \end{array}[/latex] There are eight terms in the sequence. Find the number of terms in the finite arithmetic sequence. [latex]\left\{6\text{, }11\text{, }16\text{, }...\text{, }56\right\}[/latex] In many application problems, it often makes sense to use an initial term of [latex]{a}_{0}[/latex] instead of [latex]{a}_{1}[/latex]. In these problems, we alter the explicit formula slightly to account for the difference in initial terms. We use the following formula: [latex]{a}_{n}={a}_{0}+dn[/latex] A five-year old child receives an allowance of $1 each week. His parents promise him an annual increase of $2 per week. Write a formula for the child’s weekly allowance in a given year. What will the child’s allowance be when he is 16 years old? Solution The situation can be modeled by an arithmetic sequence with an initial term of 1 and a common difference of 2.Let [latex]A[/latex] be the amount of the allowance and [latex]n[/latex] be the number of years after age 5. Using the altered explicit formula for an arithmetic sequence we get: [latex]{A}_{n}=1+2n[/latex] We can find the number of years since age 5 by subtracting. We are looking for the child’s allowance after 11 years. Substitute 11 into the formula to find the child’s allowance at age 16. [latex]{A}_{11}=1+2\left(11\right)=23[/latex] The child’s allowance at age 16 will be $23 per week. A woman decides to go for a 10-minute run every day this week and plans to increase the time of her daily run by 4 minutes each week. Write a formula for the time of her run after n weeks. How long will her daily run be 8 weeks from today? Solution Learning Outcomes Write an explicit formula for an arithmetic sequence. Write a recursive formula for the arithmetic sequence. We can think of an arithmetic sequence as a function on the domain of the natural numbers; it is a linear function because it has a constant rate of change. The common difference is the constant rate of change, or the slope of the function. We can construct the linear function if we know the slope and the vertical intercept. [latex]{a}_{n}={a}_{1}+d\left(n - 1\right)[/latex] To find the y-intercept of the function, we can subtract the common difference from the first term of the sequence. Consider the following sequence. The common difference is [latex]-50[/latex] , so the sequence represents a linear function with a slope of [latex]-50[/latex] . To find the [latex]y[/latex] -intercept, we subtract [latex]-50[/latex] from [latex]200:200-\left(-50\right)=200+50=250[/latex] . You can also find the [latex]y[/latex] -intercept by graphing the function and determining where a line that connects the points would intersect the vertical axis. Recall the slope-intercept form of a line is [latex]y=mx+b[/latex]. When dealing with sequences, we use [latex]{a}_{n}[/latex] in place of [latex]y[/latex] and [latex]n[/latex] in place of [latex]x[/latex]. If we know the slope and vertical intercept of the function, we can substitute them for [latex]m[/latex] and [latex]b[/latex] in the slope-intercept form of a line. Substituting [latex]-50[/latex] for the slope and [latex]250[/latex] for the vertical intercept, we get the following equation: [latex]{a}_{n}=-50n+250[/latex] We do not need to find the vertical intercept to write an explicit formula for an arithmetic sequence. Another explicit formula for this sequence is [latex]{a}_{n}=200 - 50\left(n - 1\right)[/latex] , which simplifies to [latex]{a}_{n}=-50n+250[/latex]. An explicit formula for the [latex]n\text{th}[/latex] term of an arithmetic sequence is given by [latex]{a}_{n}={a}_{1}+d\left(n - 1\right)[/latex] How To: Given the first several terms for an arithmetic sequence, write an explicit formula. Find the common difference, [latex]{a}_{2}-{a}_{1}[/latex]. Substitute the common difference and the first term into [latex]{a}_{n}={a}_{1}+d\left(n - 1\right)[/latex]. Write an explicit formula for the arithmetic sequence. [latex]\left\{2\text{, }12\text{, }22\text{, }32\text{, }42\text{, }\ldots \right\}[/latex] Write an explicit formula for the following arithmetic sequence. [latex]\left\{50,47,44,41,\dots \right\}[/latex] Some arithmetic sequences are defined in terms of the previous term using a recursive formula. The formula provides an algebraic rule for determining the terms of the sequence. A recursive formula allows us to find any term of an arithmetic sequence using a function of the preceding term. Each term is the sum of the previous term and the common difference. For example, if the common difference is 5, then each term is the previous term plus 5. As with any recursive formula, the first term must be given. [latex]\begin{align}&{a}_{n}={a}_{n - 1}+d && n\ge 2 \end{align}[/latex] The recursive formula for an arithmetic sequence with common difference [latex]d[/latex] is: [latex]\begin{align}&{a}_{n}={a}_{n - 1}+d && n\ge 2 \end{align}[/latex] How To: Given an arithmetic sequence, write its recursive formula. Subtract any term from the subsequent term to find the common difference. State the initial term and substitute the common difference into the recursive formula for arithmetic sequences. Write a recursive formula for the arithmetic sequence. [latex]\left\{-18,-7,4,15,26, \ldots \right\}[/latex] No. We can subtract any term in the sequence from the subsequent term. It is, however, most common to subtract the first term from the second term because it is often the easiest method of finding the common difference. Write a recursive formula for the arithmetic sequence. [latex]\left\{25,37,49,61, \dots \right\}[/latex] Explicit formulas can be used to determine the number of terms in a finite arithmetic sequence. We need to find the common difference, and then determine how many times the common difference must be added to the first term to obtain the final term of the sequence. How To: Given the first three terms and the last term of a finite arithmetic sequence, find the total number of terms. Find the common difference [latex]d[/latex]. Substitute the common difference and the first term into [latex]{a}_{n}={a}_{1}+d\left(n - 1\right)[/latex]. Substitute the last term for [latex]{a}_{n}[/latex] and solve for [latex]n[/latex]. Find the number of terms in the finite arithmetic sequence. [latex]\left\{8,1,-6, \dots ,-41\right\}[/latex] Find the number of terms in the finite arithmetic sequence. [latex]\left\{6\text{, }11\text{, }16\text{, }...\text{, }56\right\}[/latex] In the following video lesson, we present a recap of some of the concepts presented about arithmetic sequences up to this point. Solving Application Problems with Arithmetic Sequences In many application problems, it often makes sense to use an initial term of [latex]{a}_{0}[/latex] instead of [latex]{a}_{1}[/latex]. In these problems we alter the explicit formula slightly to account for the difference in initial terms. We use the following formula: [latex]{a}_{n}={a}_{0}+dn[/latex] A five-year old child receives an allowance of $1 each week. His parents promise him an annual increase of $2 per week. Write a formula for the child’s weekly allowance in a given year. What will the child’s allowance be when he is 16 years old? A woman decides to go for a 10-minute run every day this week and plans to increase the time of her daily run by 4 minutes each week. Write a formula for the time of her run after n weeks. How long will her daily run be 8 weeks from today? Contribute! Did you have an idea for improving this content? We’d love your input. Improve this pageLearn More The 'nth' term is a formula with 'n' in it which enables you to find any term of a sequence without having to go up from one term to the next. 'n' stands for the term number so to find the 50th term we would just substitute 50 in the formula in place of 'n'. There are two types of sequences that you will have to deal with: Constant Difference Sequences This is when the difference between terms is always the same. e.g. 1, 4, 7, 10, ... This has a difference which is always 3. How do you find the formula for the 'nth' term? Well, the three times table has the formula '3n' and the terms in this sequence are two less than the terms in the three times table so the formula is '3n - 2'. You can always find the 'nth term' by using this formula: nth term = dn + (a - d) Where d is the difference between the terms, a is the first term and n is the term number. e.g. 6, 11, 16, 21, ...For this sequence d = 5, a = 6 So the formula is nth term = 5n + (6 - 5) which becomes nth term = 5n + 1 Changing Difference Sequences What if the difference keeps changing? Obviously these are more difficult but once again we can use a formula! nth term = a + (n - 1)d + ½(n - 1)(n - 2)c This time there is a letter c which stands for the second difference (or the difference between the differences and d is just the difference between the first two numbers. Putting the right numbers into the formula is reasonably simple (once you've learnt the formula!). Simplifying it requires good Algebra skills so practice your Algebra! Here's an example: Here the difference between the first two numbers is 1 so d = 1 Also the second differences are 2 so c = 2 The first term is 2 so a = 2 Using the formula, nth term = 2 + (n - 1)x1 + ½(n - 1)(n - 2)x2 Getting rid of brackets (and noticing that ½ x 2 = 1): nth term = 2 + n - 1 + n2 - 3n + 2 Simplifying, nth term = n2 - 2n + 3 If all that gave you a headache there is an alternative way! 1. If the first differences keep changing but the second difference is constant then the formula is something to do with 'n2'. Make a table showing the first few terms of 'n2'. 2. In the next column of your table write the differences between the term of 'n2' and your sequence. 3. Find the formula for this new sequence using dn + (a - d) 4. Add it on to n2 to give yo your final formula. Have a look at this using the sequence above: Sequence 'n2' Difference 2 1 1 3 4 -1 6 9 -3 11 16 -5 18 25 -7 For the sequence 1, -1, -3, -5, -7 => a = 1 and d = -2 So the formula is -2n + (1 - -2) which simplifies to -2n + 3 Final Formula (Step 4) : nth term = n2 -2n + 3 (as we got before!) Try these out using both methods and decide which one you prefer.

how to find the nth term of an arithmetic series. how to get the nth term of an arithmetic sequence